**Header:**

**Independence:**

If we have two events (E1 and E2), they are independent if P(E1 and E2) = P(E1) \* P(E2)

Example:

Toss a fair coin twice

B1 = 1 if first outcome is heads, 0 else

B2 = 1 if 2nd outcome is heads, 0 else

B3 = B1 XOR B2

E1 = first bit is 1, E2 = 2nd bit is 0, E3 = 3rd bit is 0

E1 and E2 are independent

E2 and E3 are independent

Example:

Now suppose we toss 2 coins, but the first coin is biased (chances of heads is ¾)

B1 = 1 if first outcome is heads, 0 else

B2 = 1 if 2nd outcome is heads, 0 else

B3 = B1 XOR B2

E1 = first bit is 1 P(E1) = 3/4, E2 = 2nd bit is 0 P(E2) is 1/2, E3 = 3rd bit is 0 P(E3) = 1/2

E1 and E2 are still independent

P(E2 and E3) = 1/8, P(E2) \* P(E3) = ¼, so E2 and E3 are not independent

XOR of 1 pure and 1 impure random bits is pure random

IF E is an event, NOT(E) = Ω - E

Example:

Suppose we toss a fair coin n times

P(we see >= log(n)/2 consecutive heads)?